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DEVELOPMENT OF PERTURBATIONS OF A SHOCK-ACCELERATED INTERFACE BETWEEN TWO GASES

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It is known [1] that the interface between two media of unequal densities is unstable if a constant acceleration is directed from the light medium toward the heavier one. In that case small initial perturbations grow exponentially. Conversely, if the acceleration is directed from the heavy liquid toward the light one, the interface remains stable.

Richtmyer [2] analyzed the case where the acceleration is of pulse-like nature and, in particular, when the interface is accelerated by a stationary shock wave passing out of the light medium into the heavier one. In that case, for small initial sinusoidal perturbation, its amplitude grows linearly with time

$$\frac{da}{dt} = a_0\, n\, u\, R, \quad a(0) = a_0,$$

where $n = 2\pi/\lambda$; λ is the perturbation wavelength; u is the contact interface velocity; $R = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ is the Atwood number; ρ_1 and ρ_2 are the densities of the gases at the interface, compressed by the shock wave.

The experimental results [3] confirmed the conclusions [2] in a qualitative sense. In addition, the interface was found to be unstable in the case where the shock wave passes out of the heavy gas into the light one. The interface perturbation amplitude grows linearly with time in both cases, except the initial stage.

At present, there are many papers on the development of interface sinusoidal perturbations after passage of a shock wave (see, for instance, [4–9]); numerical solutions are given to two problems in [8, 10], where the development of finite "step-like" perturbations is investigated.

This paper reports the results of experimental and numerical investigations on the development of finite perturbations of various shapes. The initial perturbation amplitudes under consideration Δ_0 (being measured from an upper to a lower point of the perturbation) fall in the range $0.2\lambda \leqslant \Delta_0 \leqslant 0.8\lambda$. In the case of a perturbation $y = a_0 \cos nx$, $\Delta_0 = 2a_0$ (here Δ_0 is the "step" height).

Experimental Investigation. 1. The experiments were carried out in a shock tube using the technique described in [3].

The measuring section of the shock tube consisted of butt-joined units. A thin organic film of mass–length ratio $(3\text{–}4) \cdot 10^{-5}$ g/cm was placed at joints between the units. The units were filled with gases of various densities. The joints (and thus the interfaces between the gases) were specially shaped to obtain a certain shape of the initial interface perturbation.

The interface under study is accelerated by a plane shock wave, initiated in the channel of the shock tube. The channel of the measuring section of the shock tube has transparent walls, thus permitting observation of the flow process. The process was recorded by means of the shadow-image equipment IAB-451 combined with the optical superfast photocamera SFR-2M in the frame-by-frame mode.

2. The development of the initial perturbations of the interface between gases of various density has been studied in the experiments: air ($\rho_0 = 1.205$ g/liter, $\gamma = 1,405$) and Freon-12 ($\rho_0 = 5,13$ g/liter, $\gamma = 1,138$).

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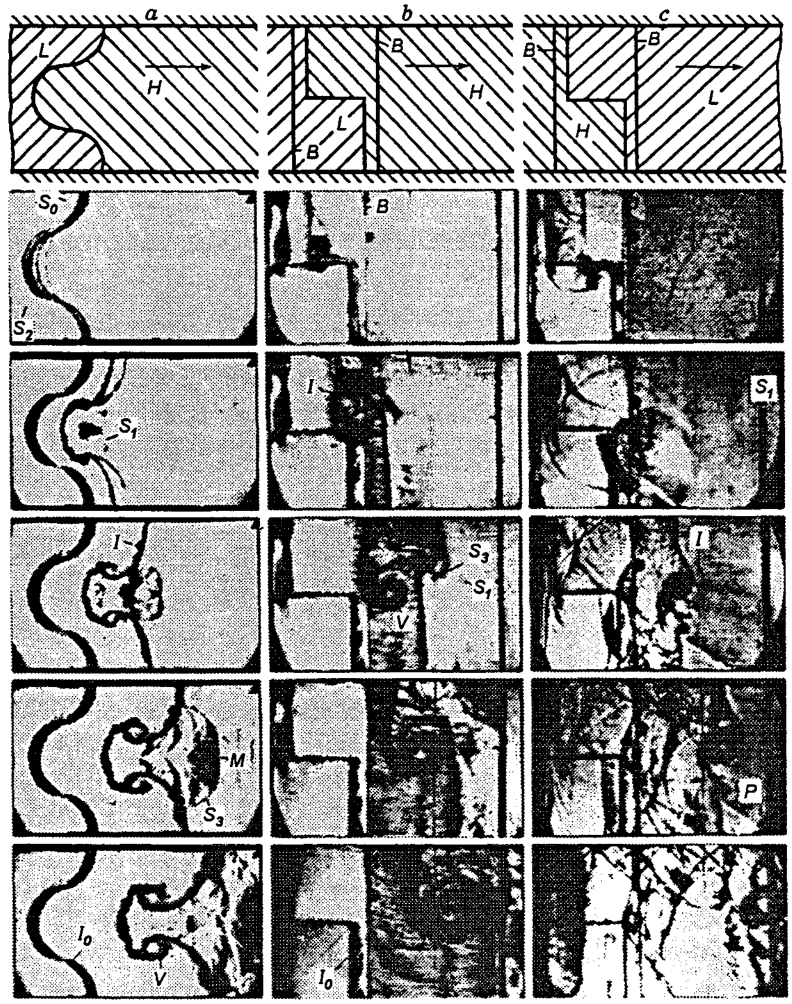

Fig. 1

Initial interface perturbations of the following types were given: 1) near-sinusoidal (conjugated circle arcs), 2) "step-like," and 3) "saw-tooth." The Mach number of the shock wave initiated in the channel of the shock tube in air is M=1.44.

3. The photochronogram frames taken in the experiments are shown in Fig. 1: *a*) a near-sinusoidal initial perturbation, $\lambda = 120$ mm, $\Delta_0 = 48$ mm, the shock wave crosses the interface passing out of air into Freon-12; *b*) "step-like" initial perturbation, $\lambda = 240$ mm, $\Delta_0 = 48$ mm, the wave passes out of air into Freon-12; *c*) "step-like" initial perturbation, $\lambda = 240$ mm, $\Delta_0 = 48$ mm, the wave passes out of Freon-12 into air. In all cases the frames are spaced 160 μsec apart.

Let us consider the case given in Fig. 1*a*, where I_0 is the initial position of the interface (the joint between units of the measuring section of the shock tube), at which a film was placed to separate the gases; I is the current interface (film) position, B is a joint in the channel of the shock tube without a film, L is the light gas, H is the heavy gas, and P is an interface perturbation.

As the shock wave S propagates through the interface I_0, the passed shock wave S_1 and the reflected shock wave S_2 originate after the discontinuity decay. Both of them are perturbed. Interaction between the incident shock wave S_1 and the inclined areas of the interface causes tangential flows with a velocity discontinuity at the interface. These flows give rise to transverse waves of compression, which transform into transverse shock waves S_3. Development of the Kelvin–Helmholtz instability at inclined areas of the interface leads to the formation of vortical zones V, which grow with time. A collision between transverse waves S in

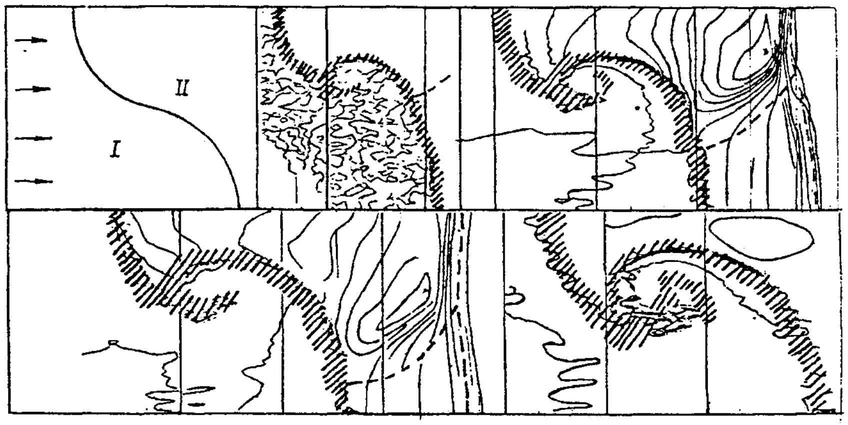

Fig. 2

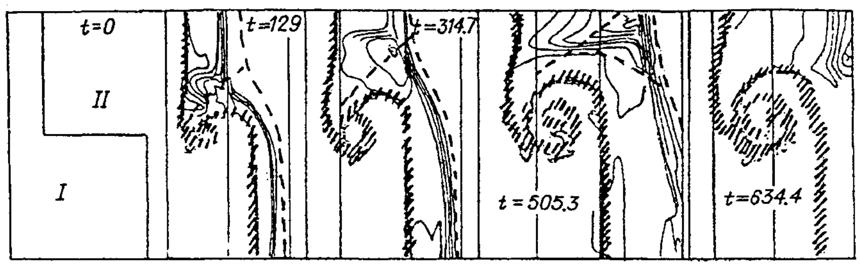

Fig. 3

the plane of symmetry leads to the formation of the Mach wave M.

4. In the case of a "step," the vortical zone begins to form almost immediately after the wave has reached the interace (Fig. 1*b,c*). The characteristic dimensions of the vortical zone are of the same order as the "step" height. In Fig. 1*b* the shock wave crosses the interface, passing out of the light gas into the heavier one, whereas in Fig. 1*c* it passes in the opposite direction. In the first case, as the shock wave crosses the interface, the perturbation amplitude ("step" height) first decreases and then grows with time, while in the second case the perturbation changes its sign and then grows in amplitude.

5. In the case of a "saw-tooth," the vortical zone does not form in the course of the experiment and the perturbation shape as a whole changes weakly, but its amplitude grows with time.

It is noteworthy that, according to [7], the amplitude growth rate could be somewhat lower in experiment because of the influence of the film's strength properties. It is shown in [7] that the growth rate of small sinusoidal perturbations in the experiment could be $\sim$ 10% lower because of the stabilizing influence of the film tension.

Numerical Investigation and Calculations. Two-dimensional gasdynamic calculations followed the techniques SIGMA [11] and EGAK [12]. In the SIGMA method the interface was taken as a calculation grid line (Lagrange line). The difference scheme of the SIGMA method has a second order of accuracy with respect to spatial variables.

In the EGAK method Euler variables are used permitting the calculation of fluxes at high deformations. To detect the interfaces, substance concentrations are used, as well as a special (donor–acceptor) algorithm of convective flux calculation in order to limit the calculational diffusion. The difference scheme of the EGAK

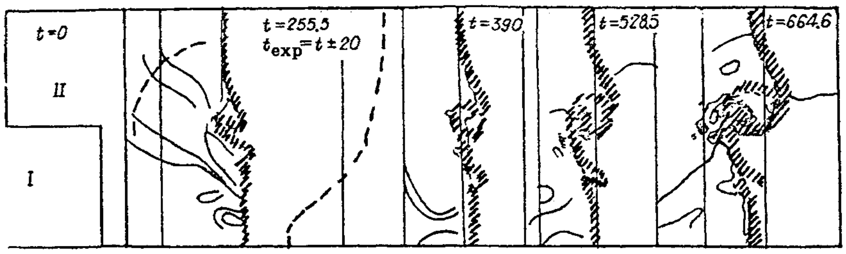

Fig. 4

method is of first order of accuracy and includes an approximational viscosity, whose coefficient is

$$\mu = \frac{h\rho u}{4}$$

(h is the size of the calculation cell).

The number of points in the calculations was chosen in such a way that the approximational viscosity has no significant influence on the development of perturbations. That question has been studied in detail in [7]. According to recommendations [7], in calculations by the SIGMA method, 20 points were taken per perturbation wavelength. The number of points in the calculations by the EGAK method was determined in the following manner.

Bakhrakh et al. [7] studied how viscosity influences the interface instability after passage of a shock wave. In the case where the first medium is a viscous liquid while the second medium is ideal, it was shown that the evolution of small sinusoidal perturbations of the interface was described by the following equation:

$$\frac{da}{dt} = a_0\, n\, u\, R\, \exp(-0.9126\, \vartheta\, n^2\, t),$$

where $\vartheta = \mu/\rho$ is the kinematic coefficient of viscosity.

By applying this formula to the approximational viscosity, we obtain an equation determining the error in the perturbation amplitude growth rate as a function of the ratio of the calculation cell size h to wavelength. Using the relation obtained, it can be shown that with 100–120 points per perturbation wavelength, the error in the amplitude growth rate in our calculations may amount $\sim$ 1% (the physical time t of the process is taken into consideration).

The equation of state of all the substances was taken in the form corresponding to an ideal gas.

Comparison of Calculated and Experimental Data. All the above experiments were calculated by the EGAK method. A comparison between the calculated and experimental data was carried out by optical projection of the frames taken in the experiment onto the corresponding plots of equidensity curves generated by a computer. Equal scaling of images and no spatial shift were ensured by superposition of the still frames taken in the experiment onto the computer plots. The error in the interface position did not exceed 2%. Figures 2–4 demonstrate the physics of the processes involved (I and II correspond to different substances). The calculated and experimental data are in agreement, except for some small-sized details of vortical zones.

Numerical Investigation of the Effect of the Initial Amplitude and Shape of Perturbations. To study the effect of the initial amplitude and shape of perturbations, several series of calculations were carried out, with varied initial shape (sinusoidal perturbation, "saw-tooth," "step") and amplitude in the range $\Delta_0 = 0.8\,\lambda$. Passage of a shock wave through the interfaces between gases [air, freon–helium ($\rho_0 = 0.167$ g/liter, $\gamma = 1.63$)] from the heavy gas into the light one and vice versa was considered. Atwood numbers are in the range $R = 0.5$–0.94.

The calculations performed allow the following conclusions:

1. The relative growth rate of the initial perturbation amplitude $(d\Delta/dt)/\Delta_0$ decreases with an increase in Δ_0/λ for all discussed shapes of the interfaces and types of discontinuity decay. For sinusoidal perturbation the relative growth rate of the amplitude ($a_0 \leqslant 0.4\,\lambda$) decreases linearly (Fig. 5).

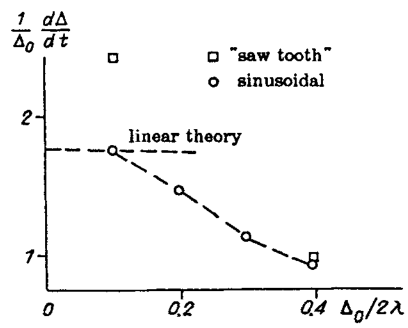

Fig. 5

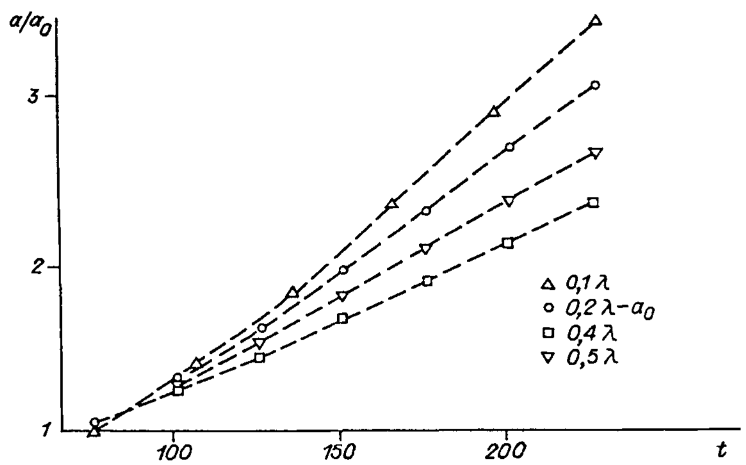

Fig. 6

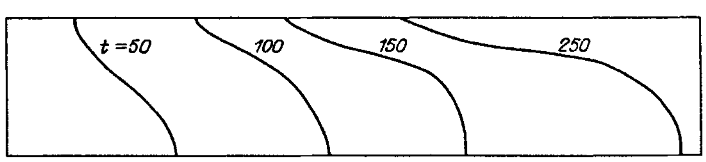

Fig. 7

2. The growth rate of the perturbation amplitude strongly depends on the interface shape, but this dependence flattens as the initial amplitude increases. At $\Delta_0 = 0.2\,\lambda$ the growth rate of the "saw-tooth" perturbation amplitude is 1.5 times greater than that of the sinusoidal one; at $\Delta_0 = 0.8\,\lambda$ the growth rates are equal in value (Fig. 5).

3. For the sinusoidal perturbations the amplitude grows linearly with time by a factor of 3–4 (Fig. 6). The shape differs considerably from the initial shape in this stage (Fig. 7). Spectral analysis of the interface shape shows that with perturbations stronger than $a_0 \geqslant 0.1\,\lambda$, the second harmonic appears almost immediately. The subsequent harmonics appear in ascending order while the preceding harmonic amplitude increased significantly. As this takes place, the expansion coefficients of the odd harmonics have the same sign as the first harmonic, while those of the even harmonics have the opposite sign. It can be shown that at $a_2 \leqslant a_1/4$ (a_1, a_2 are the coefficients of spectral expansion of the interface shape) the second harmonic does

not affect the perturbation amplitude. For the "saw-tooth" perturbation the amplitude grows linearly with time, too.

4. In numerical investigations of similar problems, special attention should be paid to the choice of number of computation points.

The evolution of a sinusoidal perturbation with the above initial amplitudes gives rise to harmonics of higher order; when the interface is of arbitrary shape, high-order harmonics are present from the very beginning. For adequate description of any harmonic of interest, the above-estimated number of computation points should be allocated per this harmonic wavelength. In this connection, calculations for convergence are of great concern for investigations of this kind.

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